TITLE

Device for the analysis of electromagnetic signals

FIELD OF THE INVENTION

5   The invention relates to the analysis of

electromagnetic signals which are a prior unknown.

In general, one is aware of how to convert a

received electromagnetic signal ("real signal") into a

complex signal, representative in amplitude and phase of

10   the electromagnetic signal received. This is done in

practice after having reduced the frequency, so as

generally to drop down to baseband. The complex signal

possesses two components, one in phase and the other in

quadrature, in the manner of complex numbers, hence its

15   name.

PRIOR ART

The techniques currently available for the analysis

of unknown signals consist in searching within the signal

20   received for the invariant attributes, by frequency

translation or level translation, in particular. They may

involve a Fourier transformation, or else a

transformation of the "wavelet" type, or else a WIGNER-

VILLE transformation, for example.

25   Although the state of the art is difficult to

perceive exactly as regards the subject-matter,

Applicants currently regard it as known practice to

search for an estimate of the mean of the "instantaneous
frequency", and to perform a demodulation of the signal
via this frequency. This could be used in the
implementation of the invention.

5

OBJECTS OF THE INVENTION

Applicants have set themselves the problem of
obtaining means which aid in the finding of the message
contained in a signal, or even make it possible to find
10   this message completely, when nothing is known regarding
the exact modulation and the alphabet of symbols used on
transmission, when for example the modulation is digital.

SUMMARY OF THE INVENTION

15   Applicants have concentrated on certain general
types of modulation, in particular on linear digital
modulation, for which types they have found solutions,
which are the constituent elements of the present
invention.

20   The invention therefore pertains, in particular, to
a device for aiding the analysis of signals containing a
symbols-based digital modulation. This device comprises:

   -   a signal memory, for storing a complex digital
signal $(z(t))$, representative in amplitude and phase of a
25   captured signal, over a chosen duration, and

   -   processing means, devised to search within the
complex signal for the properties relating to its carrier

frequency and to the modulation of the carrier frequency
as a function of a linear modulation model.

According to the invention the processing means
comprise:-

5   means for determining an estimate of the tempo (1/T)
of the modulation;

projection means effective to calculate the
components (zp(k)) of the complex signal in a function
basis ($\varphi I(t)$) which is parametrized according to the said

10   tempo (1/T) of the modulation;

and calculation means, operating on these components
so as to determine at least one estimate relating to at
least one property of the complex signal, within the
group of properties comprising the elementary pulse shape

15   (g(t)) of the complex signal, the string of symbols
(a(k)) of the complex signal and the carrier f0 used.

Advantageously the function basis is parametrized so
as to exhibit at least two samples per period (T) of the
modulation, and more generally q samples per period

20   (q ε N and q $\geq$ 2);

More advantageously the function basis comprises at
least two functions ($\varphi 1(t),...,\varphi b(t)$) which are deduced
from one another by a temporal translation of chosen
period (T/2 or more generally T/q, q ε N and q $\geq$ 2);

25   Still more advantageously the function basis can be
based in particular on rectangular functions which are

temporally adjacent to one another, or else on functions
of the "raised cosine" type.

In a currently preferred embodiment, the projection
means comprise:

5   -   means defining a digital filter, having an
impulse response substantially equal to one of the said
functions, said digital filter receiving the complex
signal, and

      -      sampling means for effecting repeated digital

10   sampling of the output of this filter at a chosen rate
(2/T; q/T, q ε N and q $\geq$ 2), depending on the type of
basis chosen.

According to another aspect of the invention, the
processing means are devised to determine an approximate

15   estimate fa of the carrier frequency f0 of the complex
signal, as well as to demodulate this complex signal
through this estimate fa. The projection means are
devised so as to operate on the complex signal (z(t))
after demodulation thereof through this approximate

20   estimate, while the said function basis is of low
frequency, substantially like the spectrum of the complex
signal which, after demodulation, is also low frequency.

According to yet another aspect of the invention,
the calculation means comprise means of matrix

25   calculation on the said components.

Preferably, a particular solution for the pair of
unknowns (gp(t), a(k)) of the model of the signal after

projection is firstly calculated in the form of a
function of minimal support (hpm(t)), together with a
symbol train (cm(k)). From this residual may be derived a
filtering residual (αi), which, associated with the pulse

5 shape of minimal support, will yield an estimate of g(t).
It will be seen that the symbols (a(k)) of the initial
signal can subsequently be derived therefrom.

DETAILED DESCRIPTION OF THE DRAWINGS

10 Expressed hereinabove in device terms, the invention
can also be defined in the form of processes.

Other characteristics and advantages of the
invention will become apparent on examining the following
detailed description, as well as the accompanying

15 drawings, in which:

- Figure 1 schematically illustrates, in block
diagram form, the principle of switching to a complex
signal (100);

- Figure 1A schematically illustrates a first

20 embodiment making it possible to implement the principle
of switching to a complex signal of Figure 1;

- Figure 1B schematically illustrates a second
embodiment making it possible to implement the principle
of switching to a complex signal of Figure 1;

25 - Figure 2A illustrates an example of an
elementary modulation function in the form of a "root of

raised cosine" pulse, as a function of time expressed in periods T;

- Figure 2B illustrates a superposition of signals a(k)g(t-kT) (or symbols) transmitted successively, as a function of time expressed in periods T;

- Figure 2C illustrates the resultant of the signals of Figure 2B;

- Figure 2D illustrates a distribution of symbols (or constellation) in the complex plane, the so-called "alphabet";

- Figure 3 illustrates the main processing steps of the process according to the invention;

- Figures 4A to 4D respectively illustrate an impulse response of a conventional real low-pass filter, a transfer function of the filter of Figure 4A, an input signal spectrum, and the spectrum of the complex signal of the filter having the transfer function of Figure 4B;

- Figures 5A to 5D respectively illustrate the spectrum of a signal obtained from a Fast Fourier Transform (FFT), a nonlinear filtering of the output of the FFT of Figure 5A, an example of the processing of the spectral lines of Figure 5B, and an example of estimating the spectral line at + 1/T of Figure 5C;

- Figure 6 illustrates in step form an example of processing making it possible to obtain the value 1/T of the isolated spectral line of Figure 5D;

-       Figures 7A to 7D respectively illustrate an
example of a signal defined by a scatter of points in the
complex plane, the spectrum of the complex signal of
Figure 7A, the spectrum of the complex signal of
5 Figure 7B after demodulation, and the spectral density of
the signal of Figure 7A before and after demodulation;

-       Figures 8A and 8B respectively illustrate the
projections of hm and of the signal in a chosen function
basis;

10   -   Figure 8C illustrates an approximation of the
functions gm(t) in the function basis chosen in
Figures 8A and 8B;

-       Figure 9 illustrates in steps form an example
of the carrying out of the processing operations aimed at
15 estimating hm(t);

-       Figures 10A to 10C respectively illustrate the
spectrum of the transfer function of the impulse response
filter for a centred signal of the type of that of
Figure 7C and for a first type of basis, the spectrum of
20 the transfer function of raised cosine functions φ, and
the spectrum of the transfer function of the impulse
response filter corresponding to Figure 10B;

-       Figure 11 illustrates the approximation of
hm(t) by its projection hpm(t) in the function basis of
25 Figure 10C;

-       Figure 12 illustrates the distribution of the
eigenvalues of the matrix ML1, arranged in descending

order of magnitude along the abscissa, when the
projection corresponds to a filtering followed by a
sampling with q = 2 points per symbol;

- Figure 13 illustrates the distribution of the
eigenvalues of the matrix ML1, arranged in descending
order of magnitude along the abscissa, when the
projection corresponds to a filtering followed by a
sampling with q > 2 points per symbol;

- Figures 14A to 14C respectively illustrate, as
a function of time expressed in periods T, the function
hpm(t), the filtering residual, and the modulated
elementary modulation function gpm(t);

- Figure 15 illustrates in steps form an example
of the carrying out of the processing operations aimed at
estimating the symbols am(k) from the possible symbol
trains;

- Figure 16 illustrates an example of dispersion
owing to the noise of the symbols a(k), in the complex
plane and for a well-centred frequency span (frequency
residual);

- Figure 17 illustrates a second example of
dispersion owing to the noise of the symbols a(k), in the
complex plane and for a poorly centred frequency span;

- Figure 18 is a three-dimensional representation
of the probability density of the symbols a(k) of
Figure 16;

- Figure 19 is a three-dimensional representation of the probability density of the symbols a(k) of Figure 17;

- Figure 20 illustrates in steps form an example of the carrying out of the processing operations aimed at estimating the position of the possible states; and

- Figure 21 illustrates in steps form an example of the carrying out of the processing operations aimed at estimating the string of symbols actually transmitted, from the train of noisy symbols a(k) and from the possible states of the symbols ei calculated with reference to Figure 20.

DETAILED DESCRIPTION OF THE PREFERRED EMBODIMENTS

The appended drawings are, in essence, of definite character. In consequence, they shall not only serve to better elucidate the description, but also contribute to the definition of the invention, as appropriate.

Elaborate mathematical expressions, or ones which are involved repeatedly, are gathered together in an appendix which forms part of the description. They are designated by the letter E, followed by a reference number.

In these expressions, the unit vector of the complex imaginary numbers is conventionally denoted j, with $j2 = -1$. Moreover, the symbol "i", employed as an index or as a summation variable, is used in several guises,

with different meanings; specifically, the sizeable number of relations invoked does not allow the systematic use of a different letter each time, and in principle the context removes any ambiguity. Likewise, the sets of

5   values taken by the variables i, k, t, ... are not generally made explicit, except when this proves to be necessary. These sets are deduced from the context by taking all the possible values over the entire duration of the signal analysed. The expressions for the minimum

10   and maximum values as a function of the unknowns of the models is of no practical interest in the algorithm. Furthermore, the symbol * as an exponent indicates the conjugate of a complex number; where a matrix is concerned, it indicates its conjugate transpose. Finally,

15   two adjacent characters may indicate a product, if the context indicates that they pertain to two different variables: for example, the product of 2 and L1 is denoted 2L1 or 2.L1.

The words "estimate" and "estimation" refer to the

20   evaluation of an unknown quantity, without any a priori assumptions regarding the accuracy of this evaluation.

Finally, to simplify the exposition and the description, the noise is not taken into account in the mathematical expressions given hereinafter.

25   The invention relates to the analysis of communication signals. Within this domain, one starts from a captured communication signal, whose raw

information content (that is to say independently of any ciphering) one endeavours to determine. It is assumed that the captured signal has been recorded and that it is analysed off-line.

5   According to the work "Digital Communications", John G. PROAKIS (chapter entitled "Representation of digitally modulated signals", p. 163, second edition, McGraw Hill Book Company), a signal corresponding to a linear digital modulation is modelled in the manner indicated in

10   relation E1 (see the appendix where most of the formulae are gathered), in which relation:

   -    the a(k) (k a positive integer) are the information symbols transmitted, which can take only a finite number of possible values, or states,

15   -   g(t) is the elementary modulation function (or "elementary pulse shape"), applied to each symbol,

   -    1/T is the tempo of transmission of the symbols, and

   -    f0 is the carrier frequency used.

20   The elementary modulation function (that is to say the one applied for each symbol) is for example a "root raised cosine" pulse shape (Fig. 2A). Such a waveform is described for example in the aforementioned work by Proakis, page 536. Usually, the elementary modulation

25   functions encountered in practice have, as in the above example, lengths greater than T. It follows that the various signals a(k)g(t-kT) (or symbols) transmitted

successively every T seconds are partially superimposed
(Fig. 2B). In accordance with the resultant of this
superposition (Fig. 2C), it is clear that this phenomenon
greatly contributes to making the analysis of the type of

5   modulation difficult.

Furthermore, the symbols used are chosen from an
alphabet or constellation of symbols (example in
Figure 2D), which are themselves expressed in complex
notation.

10   Applicants have set themselves the problem of
determining which type of modulation is employed in a
captured message, so as subsequently to determine the
parameters used for this modulation, up to the state (or
value) of the string of symbols actually transmitted.

15   The present invention aims to propose a solution
which is applicable in particular to the linear digital
modulation model alluded to hereinabove, which will be
used in the examples given hereinafter. It is recalled
that the following form part in particular of this linear

20   digital modulation model:

-    modulation by phase shift keying (PSK),

-    modulation by amplitude shift keying (ASK), and

-    quadrature amplitude modulation (QAM).

Before embarking on the detailed description, the

25   processing operations proposed will firstly be stated in
a general manner.

At the basic level, the various unknowns of the model are firstly estimated by assuming that the signal intercepted does indeed correspond to a linear digital modulation; subsequently, this assumption will be
5 validated or otherwise from these estimates.

To achieve this, Applicants have concentrated on the one hand on techniques of matrix calculation over complex numbers (linear algebra), and on the other hand on statistical aspects of signal processing.

10 The symbols contained in the starting signal are derived from an a priori unknown "alphabet" of symbols, with an elementary modulation (the modulation applied to each symbol) which is also unknown.

However, Applicants have firstly observed that, the
15 starting signal being of finite duration, the number of successive symbols which it contains is also finite. Applicants then surmised that after having assumed that this starting signal satisfies the linear digital modulation model, there must exist a means – at least –
20 of retrieving the string of symbols, by concentrating on the number of degrees of freedom (or parameters) which are necessary to obtain a representation of the starting signal (after approximate demodulation in the example described).

25 Applicants have sought such a means. This being so, they have firstly concentrated on the manner of representing the starting signal (preferably after a

first demodulation). As will be seen hereinafter, they
have found "function bases" which make it possible, by
"projection", to represent most of the signals which may
be encountered in practice, and to do so with minimal

5   loss of information, which is in this instance
negligible.

From this they have developed a technique,
nonlimiting, relying on interesting properties of a
particular solution of the linear digital modulation

10   model; for the determination of the symbols, Applicants
also exploit the fact that they are decorrelated or
weakly intercorrelated.

The said particular solution, to which we shall
return hereinafter, is unique, and makes it possible to

15   generate, through linear combinations, all the possible
solutions, as will also be seen. It is the solution hm(t)
the length of whose support is smallest.

It is known that, mathematically, the "support" of a
function f(x) can be defined as the set of values of the

20   variable x for which the function f(x) is different from
zero. Here one is dealing with functions of time
representing a signal of finite duration. Thus, the
signal z(t) has bounded support; likewise, the modulated
elementary modulation function gm(t) has bounded support,

25   which may be called the "temporal length" of the
elementary modulation. For its part, the support of the
function hm(t), that is to say the length of the time

interval over which hm(t) is non zero, is at most equal to the temporal length of the elementary modulation. This will be understood owing to the fact that it must be possible to decompose the elementary modulation according to the function hm(t).

It will furthermore be seen that, after application of this solution hm(t) of minimal support, the symbols cm(k) associated with this particular solution are expressed in a relatively simple manner as a function of the true symbols a(k) of the carrier residual $\Delta f0 = f0 - fa$ and of a filtering residual αi, and will make it possible to access these unknowns.

It would be conceivable to work on the basis of a solution other than that of "minimal support", but the expression for the residual would then be more complicated.

The various steps of the processing described in detail hereinafter are (Figure 3):

    — switch to complex signal z(t) (100),

    — estimation of the tempo 1/T (200),

    — "approximate" estimation fa of the carrier frequency f0 (300),

    — demodulation of the complex signal z(t) via this approximate carrier frequency fa (400),

    — estimation of the function hpm(t) (500, 501, 510, 520),

- estimation of the modulated symbols cm(k) associated with hpm(t) (610),

- estimation of the filtering residual αi and of the symbols am(k) from the symbols cm(k) (650),

5   - estimation of the carrier residual Δf0 (700),

- demodulation of the symbols am(k) by the carrier residual (800), so that only symbols a(k) remain,

- estimation of the possible states or values ei of the symbols a(k) with the aid of the alphabet (900),

10   - estimation of the symbol train actually transmitted e(k) as a function of the possible states (950),

- determination of the fit of the model of a linear digital modulation (990), using the parameters
15 previously estimated (validation or otherwise of the assumption).

To facilitate understanding, all the processing operations will be described, from receipt of the electromagnetic signal. However it should be clear, as
20 already indicated, that the invention can be implemented using a recorded signal which has already undergone some of the processing operations, in particular the switch to the complex signal, in which case it is already available in memory 100.

25 Figure 1 illustrates the principle of switching to a complex signal (100). The real signal received, such as it arises from an antenna 10, is defined by the

expression E1. For the processing, it is made to undergo a complex filtering 17, whose transfer function is matched to the spectrum of the signal as will be seen hereinafter. This filtering provides a calculator 20 with

5 a complex signal z(t) possessing an "in-phase" component I and an "in-quadrature" component Q. The person skilled in the art is aware that this complex signal can be represented like the complex numbers.

A complex filter centred on a frequency f1 can be

10 defined by its impulse response, which corresponds to relation E2: one starts from the impulse response p(t) of a conventional real low-pass filter (Fig. 4A), which is multiplied by the expression E3, hence the shifted spectrum of Figure 4B. The word "real" signifies here, as

15 subsequently, "real-valued", as opposed to complex-valued. Thus, with reference to Figure 1, if the spectrum of the input signal is that of Figure 4C, a filter having the transfer function of Figure 4B yields a complex signal having the spectrum of Figure 4D.

20 In one embodiment (Figure 1A) there is provided, after the reception amplifier 11, one or more intermediate frequency (IF) stages 13 followed by a baseband conversion stage 15, then by the complex filtering 17. The analog-digital conversion is generally

25 done either at the input of the stage 15, or inside the stage 15, or at its output (with a lower sampling rate in this last case).

Other techniques can be envisaged for obtaining the complex signal, in particular a Hilbert transform (not represented), or else a complex demodulation IQ into an in-phase component I and an in-quadrature component Q, as illustrated in Figure 1B: after a stage 14, where the signal lies in a band B around a central frequency f0, there is provided a stage 18, with two mixer pathways 181 and 182, receiving a signal cos 2πf0t directly or in quadrature (stage 180); the mixers are followed by filters 183 and 185, which yield the outputs I and Q, respectively. The analog- digital conversion is generally done before or after the stage 18.

Step 200 consists of an estimation of the tempo 1/T of the unknown modulation which is presumed to exist in the signal received. One of the techniques which can be used to this end will now be described.

The signal x(t) with expression E4, where the time shift τ is small compared with the value T equal to the inverse of the tempo sought, is formed at 201 (Figure 6) from the complex signal z(t). It is known that, to conjugate z(t − τ) in E4, it is sufficient to change the sign of its complex imaginary component. The time shift τ can be chosen equal to the sampling stepsize.

Applicants have noted that the sought-after tempo corresponds, in the spectrum of this signal x(t), to a spectral line of frequency (1/T). For the subsequent

processing operations, it is useful to estimate the position of this line, as accurately as possible.

To do this, the spectrum X(f) of the signal x(t) is calculated at 202. Advantageously, an interlaced (so-called "zero padding" technique, well known to those skilled in the art) Fast Fourier Transform (FFT) is used for this.

An interlacing of at least 8 (eight) is chosen as parameter of the FFT.

The output from this FFT (Figure 5A) is subsequently processed at 203, for example in accordance with relation E5: each value of X(f) obtained at a frequency f is normed via the mean of the values X(f) obtained for frequencies lying within a window of width $2.\Delta fi$, centred about f. This processing constitutes a kind of nonlinear filtering of the output of the FFT which aims to emphasize the lines contained in this spectrum (Figure 5B). The width $2\Delta fi$ fixes the degree of filtering obtained. Here, it has been fixed by simulation at 20 times the inverse of the total duration of the signal analysed.

Subsequently, at 204, this signal is thresheld in such a way as to select the spectral lines which it contains, and, at 205, the lines at the negative frequencies are ignored (Fig. 5C).

Because of the interlacing of the FFT, several sub-lines remain in the vicinity of f = 1/T (Fig. 5D). The

sought-after value 1/T is then estimated in step 206 via an interpolation which can consist in fitting a curve, of the polynomial type, for example, which passes through the tops of these various sub-lines and by taking the

5 maximum of this curve.

An approximate estimate fa of the carrier frequency f0 (step 300) is determined in parallel with the estimation of the tempo of the modulation. The approximate estimation is useful, in so far as it will

10 serve to perform a first demodulation, which makes it possible subsequently to work at low frequency: the function basis will be defined at low frequency, and the matrix calculations will be performed on samples which are likewise available at low frequency.

15 The approximate estimate fa can be obtained in various ways. Applicants preferably take fa to be the mean value of the instantaneous frequency of the signal. This mean value fa is for example calculated as follows:

- as at 201, the signal x(t) with the expression

20 E4 is firstly taken, then

- at 311, the mean value of the phase of x(t) is calculated over the entire duration of the signal, and it is divided by $2\pi\tau$.

This technique relies on the fact that, if τ is

25 small compared with T, the discrepancy between the phase of the signal z(t) at the instant t and the phase at the instant (t-τ) depends only on τ and f0. As before, τ may

be chosen equal to the sampling stepsize (case of Figure 6), or else calculated by taking a small fraction of T.

Figure 7A shows an example, where x(t) is defined by a scatter of points, corresponding to a mean value fa = 1940 Hz.

If fa has been estimated, a demodulation of the complex signal z(t) is carried out (step 400) via this approximate estimate fa of the carrier f0, in a known manner. Numerically, the demodulation consists of a complex multiplication of the signal z(t) by an expression similar to E3, where f1 is replaced with -fa.

After this demodulation, the initial spectrum (Fig. 7B) of the complex signal will centre itself approximately on the zero frequency (Fig. 7C). The residual decentring of the spectrum corresponds to a small carrier residual Δf0 equal to f0 - fa. An example is given in Figure 7D, where the chain-dotted line represents the spectrum of the signal before demodulation, whilst the continuous line represents the spectrum of the signal after demodulation.

The signal obtained after demodulation by fa will again be denoted z(t), in accordance with the linear digital modulation model. This signal z(t) henceforth satisfies formula E10. Applicants have observed that this signal z(t) can also be written according to expression E14, as a sum of products each of which involves:

- a modulated elementary modulation function gm(t), defined by relation E12, and

- a string of modulated symbols am(k), defined by relation E13.

In these expressions and subsequently, the index m signifies "modulated", and k goes from 1 to kx, which is the total number of symbols involved in the signal analysed.

This is proved by rewriting formula E10 in the manner expressed in formula E11.

It will be noted that the expression E14 is "devoid of any apparent carrier", that is to say that the carrier residual $\Delta f0$ no longer appears there explicitly. Indeed, it is contained in the "modulated" elementary modulation function gm(t) and in the modulated symbols am(k).

To access the symbols, it will be expedient to estimate this elementary modulation function gm(t). Also unknown are the carrier residual $\Delta f0$, the set of symbols used, or "alphabet", and the string or train of symbols specifically contained in the starting signal.

In the embodiment described, step 500 (501-520) consists in estimating the modulated elementary modulation function, that is to say the function gm(t). From this will be deduced the modulated symbols am(k) at step 600 (610-650) (see Figure 3).

The following difficulty arises first of all: it is a difficult task to work with a signal defined for each

value of t, given the potentially very high number of values to be considered.

Applicants propose that an approximation be made by projecting the signal z(t) onto a "basis" of functions, which is chosen in advance. This basis (in the mathematical sense of the word) is in fact a free family, preferably orthonormal, of functions ($\varphi_1(t)$, $\varphi_2(t)$, ... $\varphi_b(t)$).

A function basis (φ_1, φ_2, ...φ_b) matched to the tempo $1/T$ as estimated earlier is chosen and constructed for example in the manner indicated in expressions E25 and E26. In these expressions, rect() denotes the rectangular function over the time interval indicated below. The integer q is such that the product q.T is substantially equal to the length of the signal analysed. b corresponds to the number of bases functions required to cover the entire duration of the signal analysed.

Other conceivable function bases will be indicated later. However, Applicants currently consider that the function basis of expressions E25 and E26 remains the simplest, and yields approximations which are amply sufficient for signals encountered in practice.

Whatever embodiment is chosen, a "representation" of the function basis exists in the device. It is this that is recalled by the block 501 of Figure 3. In this instance, this representation can be parametrized by T.

In the embodiment described in detail hereinafter, the basis defined by E25 and E26 is used. There are thus two basis functions per interval of T seconds: thus two projection coefficients will be obtained every T seconds (one also talks about "two points per symbol", given that a symbol is received every T seconds). The way in which the invention can be extended to integer values greater than 2 ($q \geq 2$) will be seen later.

The approximation then begins with the calculation of the projection of the signal $z(t)$ in the space (likewise in the mathematical sense of the word) generated by the functions $\varphi i(t)$. This projected signal will thereafter be defined only by projection coefficients which, themselves, will be finite in number, less than the number of time samples of the signal $z(t)$.

The signal $z(t)$ is therefore firstly projected (510, Fig. 3) onto the function basis defined by relations E25 and E26. This projection is denoted $zp(t)$ where the index p signifies "projected".

Applicants have observed that approximating the signal $z(t)$ by its projection in the basis consisting of the $\varphi i(t)$ amounts to approximating the functions $gm(t-kT)$ (k varying) involved in the model of the signal by their projections in this same "basis". The basis is chosen so that all the basis functions are deduced by translation by an integer number of T of a small number of functions $\varphi 1(t)$, $\varphi 2(t)$, ... (two functions for a basis having two

points per symbol, q functions in the general case).
Applicants have then observed that the projections of the
functions gm(t-kT) are deduced from one another by
translation by T: they are therefore defined by the same

5   projection coefficients to within a time shift.
Everything takes place as if the unknown function gm(t)
had been approximated by its projection in the basis
φi(t), which is written with a small number of unknown
coefficients, which will need to be found.

10   For greater simplicity, the functions φ1(t), φ2(t),
... generating the basis by translation by kT (k ε N) are
chosen in such a way that they are also deduced by
translation by T/2 (more generally by T/q, q ε N) of one
and the same function φi(t). We thus obtain the bases E25

15 and E26 (two points per symbol) and E34 and E35 (q points
per symbol), where the functions φi(t) are ultimately
deduced from one another by translation by kT/2 and kT/4
respectively (k ε Z).

    Applicants have moreover observed that the functions

20 gm(t) encountered in practice are sufficiently smooth as
to be approximated with few errors in these function
bases. A schematic example of such an approximation is
illustrated in Figure 8C.

    Applicants have also observed that, in order to

25 project the signal z(t) into the function basis defined
by relations E25 and E26, it is possible to filter the
signal via the filter of impulse response φ1(-t)

(expression E25), then to sample the output of this filter every T/2 seconds. It is therefore apparent that this projection is particularly simple to undertake.

As shown in Figure 10A, the signal is centred on the zero frequency and possesses, in practical cases, a spectral band of width 1/T. The projection filter is also centred on the zero frequency; by contrast, it possesses a band of 2/T (at 4 dB). The person skilled in the art will deduce therefrom, as has already been stated in respect of the function gm(t), that, as a result of the approximation made by this projection, that part of the signal which is lost by projection is negligible.

Incidentally, interesting synergies of this embodiment will be noted: the prior estimation of the tempo enables the projection of the signal onto the function basis to be well synchronized with the modulation tempo; estimation of the mean of the instantaneous frequency enables this projection to be done on a function basis of low frequency.

Work by Applicants has shown that with two samples per symbol (at least), that is to say with at least two projection functions every T seconds, it becomes possible to find solutions of the model (linear digital modulation model) which correspond to the starting signal, with the aid of matrix calculation techniques (block 520, Fig. 3).

However, the said work has also shown that, for a given starting signal, relation E14 admits a multiplicity of pairs of possible solutions.

Among all these possible solutions, Applicants have
5 focused on searching for that which satisfies the following property: we require the solution yielding modulated symbols am(k) which, after demodulation, will take the smallest possible finite number of values.

After having envisaged imposing this constraint
10 directly, Applicants currently prefer to proceed in two stages, as will be seen.

At least when z(t) is a signal arising from a linear digital modulation, Applicants have shown the following:

- among the many possible solutions of the
15 modelling, there exists only one (cm(k), hm(t)) satisfying relation E20, where hm is, among the solution modulation functions, the one whose support is of shortest length;

- furthermore, this particular solution (cm(k),
20 hm(t)) generates all the possible solutions as follows: if (am(k), gm(t)) is another solution, then there exist n coefficients (α1, ...αn) making it possible to satisfy expression E21 yielding gm(t), whilst the symbols am(k) are defined recursively, for all the values of k, by
25 expression E23.

Applicants have derived therefrom that the modulated elementary modulation function gm(t) can be written

according to expression E21, where hm(t) is the unique solution function of smallest support.

Basing themselves on the foregoing remarks, Applicants advocate that the search for the function gm be split into two parts:

- initially, one searches for the solution (cm(k), hm(t)) with the constraint that hm(t) has the smallest possible support,

- subsequently, utilizing the symbols cm(k) corresponding to this solution hm, and modelling according to expression E23, one estimates the second part of the function gm, namely the coefficients (αi), which is globally dubbed the "filtering residual".

Subsequently the signal will be modelled by the pair of relations E22 and E23, where hm(t) is the solution function of smallest support, and the (αi) are the filtering residual.

As was stated previously, approximating the signal z(t) by its projection in the space of functions φi(t) amounts to approximating the function gm(t) by its projection in this same space. The same holds for the particular solution hm(t), which will be replaced with its projection in the basis of functions (φ1(t), φ2(t), ..., φb(t)). This projection hpm(t) will thereafter be defined only by a small number of projection coefficients.

Now, the projection zp(t) of z(t) is hereafter
modelled only using the projection hpm(t) of hm(t). The
(small) part of the signal which is orthogonal to the
projection "basis" is lost. The function basis E25, E26
5   has been chosen in such a way that the lost part of the
signal is sufficiently weak for the signals encountered
in practice.

An example of the carrying out of the processing
operations aimed at estimating hm(t) will now be given,
10   with reference to Figure 9.

Firstly, as already indicated, the signal z(t) is
projected onto the function basis $\varphi i(t)$. In this
instance, this consists, at 510, in filtering the signal
z(t) through the filter of impulse response $\varphi 1(-t)$
15   (expression E25), then, at 523, in sampling the output
from this filter every T/2 seconds, up to the end of the
signal z(t).

The function hpm(t) of smallest support, which is
the solution of equation E30, is then estimated.

20   The projection zp(t) of the signal z(t) can be
written using its projection coefficients z(k) according
to relation E36 (with p standing for projected).

For its part, the projection hpm(t) of the function
of smallest support hm(t) is written according to
25   relation E37, where L is the length of hpm(t), reckoned
in terms of number of periods T. Stated otherwise, the

length of hpm(t) is greater than (L-1).T and less than or equal to L.T.

Given the rate of two samples (two projection coefficients) per symbol every T seconds, two successive samples of rank 2k + 1 and 2k + 2 of the projection zp(t) correspond to the pair of values z(2k+1) and z(2k+2), which pair is modelled according to relations E38.

The first step of the processing consists in estimating the length L of hpm(t).

In this regard, Applicants have noted that the lengths of the elementary modulation functions gm(t) encountered in practice are generally less than 10 times T.

A parametrizable upper bound L1, such that L1 = 10.T, is then chosen. With each rank k, step 525 matches up a vector or unicolumn matrix ZL1(k) in accordance with expression E41. The vectors ZL1(k) correspond to signal slices z(t) taken between instants kT and (k + L1)T. This vector ZL1(k) contains 2.L1 successive components in r(k), going from (2.k+1) to (2.k+2.L1). The vectors ZL1(k) overlap one another in large part: ZL1(k+1) is deduced from ZL1(k) by eliminating z(2k+1) and z(2k+2) from the top, by raising the other elements by two slots, and by adding z(2k+2L1+1) and z(2k+2L1+2) at the bottom. The construction of these vectors ZL1(k) from the basic data is therefore done with an interlacing.

The square matrix ML1, of dimension 2.L1, defined by relation E42, is formed at 527. This is a kind of covariance matrix of the quantities z(k); however, in contradistinction to the customary covariance, the constituent vectors are interlaced; this is why Applicants propose to dub ML1 the "interlaced covariance matrix".

The eigenvalues ML1 are calculated at 529 in a known manner. Their analysis makes it possible to access the length L of the function hpm(t), since Applicants have established that the matrix ML1 possesses L1 + L − 1 non-zero positive eigenvalues, and L1 − L + 1 (less than L1 + L − 1) zero (in fact almost-zero on account of noise) eigenvalues.

For example, these eigenvalues can be arranged in descending order of magnitude. The distribution of the eigenvalues is given approximately by Figure 12, where the ordinate is the magnitude of the eigenvalues, which are arranged in descending order of magnitude along the abscissa. The index or rank of the eigenvalue (after sorting) is estimated, this corresponding to the break point of the line of Figure 12, that is to say to the point where one drops abruptly to almost-zero eigenvalues, corresponding to noise. This break point is conventionally estimated by using the so-called "MDL" (standing for Minimum Description Length) criterion. Refer to the article "Detection of Signals by Information

Theoretic Criteria", IEEE Transactions on Acoustics, Speech and Signal Processing, April 1985, p. 387. The estimation of this break point can be interpreted as the estimation of the rank of the matrix ML1.

The length L of the function hpm(t) is given by the post-sorting rank of the first zero eigenvalue.

Vectors ZL(k) defined by the 2.L consecutive components in z(k) of expression E44 are now formed at 535. The matrix ML (dimension 2L × 2L) with expression E43 (similar to the expression E42, but with L instead of L1) is formed, at 537, from these vectors ZL(k).

This matrix ML is another "interlaced covariance matrix", but with an interlacing restricted to L instead of L1. The vectors ZL(k) correspond to slices of the signal zp(t) which are taken between the instants kT and (k + L)T.

The processing is then as follows. The smallest eigenvalue of ML is determined at 539 by calculating, for example, as before, all the eigenvalues (which are positive on account of the hermitian symmetry of ML) and by arranging them in descending order.

The eigenvector associated with this smallest eigenvalue is calculated at 541. Applicants have in fact established:

     —   that in this case the matrix ML has a single almost zero eigenvalue λ,

- that the magnitude of this eigenvalue makes it possible to estimate the power of the noise $\sigma 2 = \lambda/L$, and

- that the eigenvector associated with this eigenvalue (expression E45) depends only on the function hpm(t), the solution having the smallest support.

The projection coefficients of hpm(t) are therefore obtained (apart from the sign) at 543 by taking the complex conjugates of the components of this eigenvector in order, and by changing the sign of one out of every two of these components (one can work to within the sign).

It will be noted in passing that steps 535 to 539 are the same as 525 to 529, on replacing L1 with L. They can therefore be implemented by the same calculation means.

This processing can be interpreted as follows:

- the vectors ZL1(k) correspond to a chopping of the signal zp(t) into slices of length L1.T, which overlap;

- these vectors are defined by the 2.L1 projection coefficients $(z(2k + 1),...,z(2k + 2L1))$, which are a priori capable of taking any value in the corresponding vector space of dimension 2.L1;

- however, on each slice this signal zp(t) becomes the signal u(t) according to relation E47, whose elements $z1(t),...,zL+L1-1(t)$ are defined by relations E48;

- consequently, on each of these slices, the signal zp(t) is always a linear combination of the same (L1 + L − 1) signals z1(t),...,zL+L1-1(t), a linear combination whose coefficients are the symbols cm(k) involved in the relevant slice and therefore varying from one slice to another;

- although defined at the start using its 2.L1 projection coefficients (z(2k + 1),...,z(2k + 2L1)), each slice of the signal zp(t) therefore in fact takes its values only in the same function subspace generated by the L1 + L − 1 signals defined hereinabove.

Applicants have also shown that the condition "hpm is the solution having the smallest support" is equivalent to the condition "the previous L1+L-1 signals are linearly independent".

On each slice of length L1T, the signal zp(t) therefore takes its values in a space of dimension L1+L-1.

The matrix ML1 defined previously makes it possible, by diagonalization, to estimate the subspace containing these signal slices, from the eigenvectors associated with the non-zero eigenvalues. To simplify the processing, it is possible, as described, to estimate only the rank L1+L-1 of this matrix ML1 (number of eigenvalues which are not very small or zero). After having deduced L therefrom, it is then possible to rechop the signal into slices of length L.T.

The same reasoning as before then demonstrates that these signal slices generate, in the function space of dimension 2.L, a subspace of dimension 2.L-1, since:

- it is now known that each of the slices of length L.T of the signal zp(t) is capable a priori of taking its values in a space of dimension 2.L; but that in fact

- on each of these slices, the signal zp(t) is always a linear combination of the same (2.L-1) signals s1(t)...sL+L-1(t) of equation E48 where L1 is replaced with L.

In short, there exists, in the function space, a direction which is orthogonal to each of the signal slices: this is the one generated by the eigenvector V of expression E45, associated with the zero eigenvalue of the matrix ML.

hpm(t) has now been estimated. It is then possible to go to step 600, which is the estimation of the train of modulated symbols cm(k) associated with this estimated function hpm(t).

This is done by filtering the signal zp(t) through the inverse filter 610 of hpm(t). A nonlimiting way of calculating the inverse filter will now be indicated, utilizing the pseudo-inverse matrix of the filtering matrix associated with hpm(t).

One starts from the expression for hpm(t) given in E58 which is the result of the previous processing,

obtained via its extension for a projection having more than two points every T seconds. For the particular case of two points per symbol, dealt with hitherto, it suffices to put q = 2.

5   The matrix H given by expression E63 is then formed. This is a rectangular matrix of dimension qL.(L1 + L-1) which may be called the "filtering matrix". It is formed as follows. Each column possesses q.L components. The first column is obtained by taking the last q components

10   of hpm(t), and by supplementing with zeros. For the remainder, each column is deduced from the previous one by a downward shift of q slots, whilst supplementing on top with the components of hpm(t) in order, then with zeros, when these components have run out. One continues

15   in this way until the last column containing only zeros at the top and then the first q components of hpm(t) at the bottom. A matrix with L1 + L - 1 columns is thus obtained.

One then forms the matrix product of expression E64,

20   where the person skilled in the art will recognize the pseudo-inverse matrix of H, of dimension (2.L-1).qL. The coefficients of the inverse filter are estimated via row number L of the matrix given in E64.

The signal zp(t) is then applied at 610 to the

25   inverse filter thus obtained. The output corresponds to the symbols cm(k) of expression E43, in which αi is the filtering residual and the am(k) correspond to the

sought-after symbols modulated by the carrier residual $\Delta f0$, all these quantities being still unknown at this stage of the algorithm.

To estimate these unknowns, as has been seen, Applicants have envisaged imposing the following condition: the symbols am(k), associated with the filtering residual (αi), must correspond to modulated symbols which, after demodulation, will take the smallest possible finite number of values. The condition which Applicants prefer to impose today is: "the symbol train (am(k)) is, from among the possible solutions yielding decorrelated symbols, that for which the alphabet (set of all possible values) is of smallest size". The condition that the symbols are decorrelated is given by relation E70 and pertains to the covariance of the symbols. The second condition consists in fact in choosing from among the solutions making it possible to overcome the first constraint, the symbol train for which the variance of the modulus of the symbols is smallest (equation E80).

These two constraints, which are simpler to implement, lead substantially to the same result for the signals encountered in practice. (The true symbols transmitted are generally decorrelated).

The individual elements (αi) of the filtering residual are obtained at 650.

In step 651 (Figure 15), Applicants start from a polynomial $Q(Z)$ constructed from measures in accordance with expression E73, where Z is the variable. Next:

- at 653, there are calculated the N complex roots β_i of this polynomial $Q(Z)$ with modulus less than 1,

- at 655, there are constructed the 2^N different possibilities of N-tuples constructed from the N roots β_i, by replacing, or otherwise, each root β_i by its conjugate inverse $1/\beta_i^*$. Each of the N-tuples is then interpreted as the roots of a filter,

- at 657, there are thus obtained 2^N possible filters one of which corresponds to the filtering residual α_i which we seek to calculate,

- then, at 658, for each filter possibility thus obtained, there is estimated from the symbols $cm(k)$ the train of symbols $am(k)$ corresponding thereto (by inverse filtering) in accordance with relation E23.

It then remains at 659 to choose, from among these 2^N possible symbol trains, the one which corresponds to symbols $am(k)$, which, after demodulation by the carrier residual $\Delta f0$ which is still unknown at this stage, will take the smallest possible number of different values. One way of proceeding then consists in determining, from among all the possible symbol trains thus identified, the one for which the mean variance of the modulus of the symbols (still modulated) is smallest. This variance is

defined by relation E80, in which nb denotes the number of symbols.

The basis of this processing will now be explained.

With the filtering residual (αi) there can be

5 associated a polynomial P corresponding to its Z transform the form of which is described by expression E71, where N is an upper bound of the number n of coefficients αi. Specifically, this number n is not known in advance but, for the signals encountered in practice,

10 an upper bound N for it can be found. It has already been seen, in fact, that the elementary pulse shapes g(t) encountered in practice have a length of less than 10.T. According to expression E12, the same therefore holds for gm(t). By using expression E21, it is therefore possible

15 to deduce from it that m is less than 10 (the length of hm(t) is in terms of number of T at least equal to 1). In practice N = 10 is taken as upper bound, but this value can be parametrized in the algorithm. The only effect of taking an upper bound is that the last few coefficients

20 αi with $i \in \{n+1,\ldots,N\}$ are zero, if this upper bound N turns out to be greater than n.

Moreover there is defined the polynomial P2(Z) with the expression E72, corresponding to the modulus squared of the polynomial P(Z), and which can be interpreted as

25 the Z transform of the modulus squared of the transfer function of the filtering residual αi. It is also possible to write the polynomials P(Z) and P2(Z) in

factorized form, according to expressions E75 and E76, where the βi are the roots of the polynomial P.

Applicants have shown, by using the assumption that the symbols are decorrelated, that the polynomial P2(Z) can be estimated by the aforesaid polynomial Q(Z), to within a multiplicative constant. This polynomial Q(Z) can itself be interpreted as an approximation of the Z transform of the autocorrelation function of the symbols. They have also observed that:

- if βi is a root of P2 than $1/\beta i^*$ is also a root of P2,

- if βi is a root of P2 then βi or $1/\beta i^*$ is a root of the polynomial P which one is seeking to estimate.

In a variant, it is possible to extend this processing to the case of symbols which might be slightly correlated, that is to say which may be correlated but only with their close neighbours, in accordance with relation E81. In this case, Applicants have observed that a root βi of Q(Z) will not always be a root of P2(Z), which serves to define the roots of P(Z).

To form the various possibilities for P, for each root βi of Q with modulus less than 1, there will now be a choice between taking βi or taking $1/\beta i^*$, or taking neither βi nor $1/\beta i^*$. If N is the degree of the polynomial Q, there will therefore be 3N possible filters for choosing the filtering residual. The choice of the

correct solution is then made as earlier, by minimizing the variance of the modulus of the symbols.

Exemplary results from all the processing operations for estimating gpm(t) will now be examined:

- Figure 14A illustrates the function hpm(t);

- Figure 14B illustrates the filtering residual, where spikes corresponding to the coefficients αi stand out;

- Figure 14C illustrates the modulated elementary modulation function gpm(t); and

- Figure 17 illustrates the symbols am(k) output by the processing, which correspond to the sought-after symbols, but remain modulated by the carrier residual $\Delta f0$.

These symbols are modelled by the expression E13. The carrier residual $\Delta f0$ now needs to be found so as to deduce therefrom the true symbols a(k), fully demodulated. The processing 700 consists in testing all or part of a span of frequencies [-f1, f1], discretized with a sufficiently fine stepsize. For each tested value of the carrier residual, we calculate the probability density d(x,y) of the corresponding symbols according to formula E82, or else some other quantity of like properties. Refer to the article "an estimation of a probability density function and mode", E. Parzen, Annals of Mathematical Statistics, 1962. The criterion used to select the correct value consists in choosing the

frequency $\Delta f0$ such that expression E85 is a maximum. The maximization of this criterion may be likened to minimization of the entropy (measure of disorder) of the symbols obtained after demodulation. It would however be

5   possible to replace this criterion with other functions of the entropy.

When the frequency tested is close to the correct value (Figure 16), the distribution in the complex plane of the symbols a(k), which is deduced therefrom by

10   demodulation by the frequency tested, corresponds to a constellation with a small number of states, hence a "concentrated" probability density, which is better perceived in the three-dimensional representation of Figure 18.

15   When the frequency tested is far from the correct value, the corresponding distribution of the symbols a(k) is more spread out according to circles centred on the origin (Figure 17, where the scale deforms the circles into ellipses). Figure 19 shows this in a three-

20   dimensional view.

Once the value of $\Delta f0$ has been estimated in this way, a demodulation 800 makes it possible to obtain the symbol train a(k) corresponding thereto. This demodulation consists of a complex multiplication of the

25   signal by an expression similar to E3, where f1 is replaced with $-\Delta f0$.

The possible states of the symbols must still be estimated (step 900).

This step takes account of the fact that the symbols obtained after demodulation correspond to the true symbols, with a number of finite states, but corrupted by the noise of power $\sigma2$, calculated previously, as shown in Figure 16 which corresponds to an exemplary result obtained. The various values of the symbols a(k) found are plotted in the complex plane. Figure 2D corresponds to the alphabet of the symbols a(k) actually transmitted for this example. The phase rotation between the two Figures as well as the different scales stem from the fact that, everything being unknown at the start, the symbol train can only be retrieved to within a multiplicative complex constant.

The objective of this part of the processing is to estimate the position of the possible states. The processing is carried out as follows (Figure 20):

–   the first possible state e1 is estimated as the (x,y) position of the maximum of the probability density d(x,y) of the noisy symbols,

–   from this density, the function d2(x,y) corresponding to d(x,y) is constructed but the value is set to zero for all the points lying within a radius of 3σ (for example) around e1,

- the second possible state e2 is then estimated by taking the (x,y) position where the function d2 is a maximum,

- the processing is continued in this way until all the possible states have been processed.

This can be achieved according to steps 901 to 909 of Figure 20. Among various possibilities, the stoppage criterion for this processing ("COND" condition at 907), making it possible to ascertain whether all the possible states have been estimated, may be that of expression E86, where λ is a threshold which is fixed in advance $(0 < \lambda < 1)$.

The principle used is as follows. It is assumed that the values of the symbols transmitted are equiprobably distributed between the various possible values of the alphabet (there is almost the same number of symbols in each of the modes of Figure 18, for example). The density integrals of expression E86 are proportional to the numbers of symbols forming these densities. Assume that one has estimated n - 1 states e1,...,en-1. The stoppage criterion consists in calculating a measure of the mean number per state of symbols corresponding to the states e1,...en-1 (right-hand side of E86) and in comparing this quantity with the measure of the number of remaining symbols. If the latter quantity is too small, then not enough symbols remain to form an additional state. These

symbols are assumed to be far from the estimated states e1,...,en on account of the noise spikes.

It is now possible to proceed with the estimation of the symbol string such as it was actually transmitted (step 950).

This part of the processing consists, using the train of noisy symbols a(k) and the possible states of the symbols e1, e2,... calculated previously, in estimating, for each noisy symbol a(k), the state e(k) intentionally transmitted.

For this purpose, for each symbol a(k), starting from k = 1 (step 951, Figure 21), one searches for the possible state ek which is closest, for example (953) on the basis of the euclidian distance (DIST[] function). There will thus be obtained, for rank k, a symbol denoted e(k) taking one of the values ei of the previously estimated alphabet. The minimum distance is determined at 955 in accordance with relation E87, and one can continue by incrementing k (959) up to the maximum value kx (957), in fact until all the noisy symbols a(k) obtained at 800 have run out.

The processing operations for estimating the unknown parameters of the model E1 of the signal, under the "linear digital modulation" assumption, have finished. It now remains to verify that the basic assumption is correct, stated otherwise to validate – or otherwise –

the assumption that a linear digital modulation is present (step 990).

The previous processing operations made it possible to estimate the elements recalled at E91. The quantity defined by expression E92 is now calculated and this quantity is compared with a threshold corresponding to the allowable power of the noise. This allowable power corresponds to the noise power $\sigma2$ calculated in the "calculation of hpm" part (520), multiplied by the ratio of the starting signal sampling frequency to the tempo of the projection (q/T), and weighted by a coefficient β $(0 < \beta < 1)$ so as to take account of the imperfections in the previous estimates of the parameters. The value of β is fixed by simulation.

In short, when the signal received actually corresponds to a linear digital modulation, the difference between the signal received and the estimated linear digital modulation must be a signal corresponding to noise, of low power; conversely, when the signal received is not a linear digital modulation, the said difference is then no longer a signal of low power which may be viewed as noise.

The processing operations described may be implemented on a workstation of the Pentium PC type, with software tools suitable for calculational processing operations, such as MATLAB software. The programs prepared under MATLAB can be used as is, or better

transformed into a programing language which executes faster, such as the C language. Utilities for converting MATLAB into the C language are available for example from the distributors of the MATLAB software.

5   In certain cases where several signals, determinable by a priori spectral analysis, exist in the frequency band analysed, it may be necessary to process the signal received by parts, to each of which will be assigned its own estimate of the carrier frequency.

10   The basis of "rectangular" functions which was described above is not limiting. Recourse may be had to other sets of functions which satisfy the condition for defining a basis, not necessarily orthonormal, provided that the correlation induced with regard to the

15   projection coefficients is taken into account.

It remains especially beneficial to use a basis whose functions are deduced from one another by time shifting. This in fact permits an advantageous realization of the projection in the form of filtering,

20   followed by a sampling. This possibility is worthwhile for other types of function basis:

        —   if for example $\varphi 1$ is taken to be a raised cosine function of spectral width $2/T$ (the other functions φi being deduced therefrom as before), the

25   spectra are then those of Figure 10B. The person skilled in the art will understand that, owing to the rectangular

shape of the spectrum of φ1, still less signal is lost by the projection;

- the function basis of relations E34 and E35 can also be taken. This amounts (Figure 10C) to filtering the signal through the new function φ1(t) and to sampling every T/q (sampling with q points per symbol, with q > 2). As shown by Figure 11, in comparison to Figure 8C, the approximation of hm(t) by its projection hpm(t) in this "basis" is better.

Processing by interlaced covariance matrices was proposed above, applied to the function basis of relations E25 and E26. This processing remains applicable with any function basis for which the projection can be interpreted as a filtering followed by a sampling every T/2 (that is to say with 2 points per symbol).

On the other hand, when the projection corresponds to a filtering followed by a sampling with more than 2 points per symbol, Applicants provide for another version of the said processing, which will now be described, denoting by q the number of points per symbol, with q > 2 (or q = 2).

After projection, the new measures z(k) (signal projection coefficients) are modelled in the form of relations E50. The vectors ZL1(k) are defined according to relation E51 (like E41, but with q in place of 2).

The matrix ML1 can be formed and processed as before. It is given by expression E52. For the same

reasons as before, this matrix ML1 possesses p.L1

eigenvalues of which L + L1 - 1 are non-zero. The

distribution of the eigenvalues is that of Figure 13. The

break in the chart makes it possible here also to

5   estimate the length L (in terms of number of T) of the

function hpm(t).

The difference comes in subsequently: if, as before,

knowing L, we were to calculate the matrix ML, it would

turn out that this matrix, instead of a single zero

10   eigenvalue, would have q.L - (2.L - 1) zero eigenvalues

(p.L eigenvalues in all, of which 2L - 1 are non-zero,

following the reasoning given above). When q = 2, we have

2L - (2L - 1) = 1 zero eigenvalue. hpm(t) is deduced from

the unique eigenvector associated with this eigenvalue.

15   When q > 2, there exists several eigenvectors associated

with the zero eigenvalues. It is then no longer possible

to deduce hpm(t) therefrom in a simple manner.

To estimate the coefficients of hpm(t), Applicants

then make provision to calculate and to use the

20   eigenvectors Vi, from VL1 + L to Vq.L1, associated with

the zero eigenvalues of the matrix ML1. The q.L1

components of such a vector (with index i) are denoted in

the manner indicated in E55, with I ε {L1 + L, ...,

p.L1}.

25   One then forms the matrices Pi defined in E56. These

are rectangular matrices of dimension q.L * q(L1 + L)-1

formed in the same way as the matrix H making it possible

to generate the filter inverse to hpm(t). Each column of such a matrix possesses q.L components. The first column is obtained by taking the last q components of the relevant vector Vi, and by supplementing it with zeros.

For the remainder, each column is deduced from the previous one by a downward shift of q slots, supplementing the top with the components of Vi in order, then with zeros when these components have run out. Thus, in the second column, we take q extra components at the top, and so on until the column is completely filled with zeros and the first q components of Vi. The number of rows of the matrices Pi being fixed at q.L, we obtain L1 + L - 1 columns.

One next forms the matrix G defined by expression E57. It can be regarded as an approximate form of a so-called "projection into the noise space" matrix.

One searches for the eigenvalues of the matrix G, then for the eigenvector associated with the smallest eigenvalue. The components of this eigenvector are, in order, the sought-after components of hpm(t) in the relevant function basis.

Indeed, Applicants have shown that this matrix has a single zero eigenvalue (no noise), and that the eigenvector associated with this eigenvalue obeys relation E58.

The power $\sigma 2$ of the noise is in this case estimated by taking the mean of the almost-zero eigenvalues,

divided by L1. The symbol train cm(k) is itself estimated
from zp(t) and hpm(t) in the same way with q > 2, as with
q = 2, by constructing the matrix H in accordance with
expression E63, taking the value q used.

5   This process is preferential in the case where there
are more than 2 points per symbol (q > 2). It can also be
applied in the case where q = 2. In this case hpm(t) is
estimated directly from ML1 without involving ML.

The invention is not limited to the embodiments
10   described.

For certain applications, it may be advantageous to
refine the approximate estimation fa of the carrier
frequency f0. Various techniques are available for this
purpose. It is for example possible to estimate fa by
15   searching for the frequency, which, after demodulation of
the signal thereby, minimizes the quadratic band of the
signal. If one succeeds in rendering the residual
discrepancy negligible, then the step for estimating $\Delta f0$
and the demodulation which follows it become irrelevant.

20   It will be noted that the starting signal captured
and recorded cannot always be likened to the signal
actually transmitted, from which it may differ for
various reasons, for example multiple propagation paths.
This may result in alterations to certain steps of the
25   processing. Multiple paths, which may be pictured as
echoes, modify the shape of the function g(t) and more
especially increase its length. The modification of the

shape does not change the algorithm since the latter estimates this shape without any a priori assumptions. On the other hand, the upper bounds L1 and N of the lengths of hpm and αi must be increased. These quantities which

5 can be parametrized within the algorithm have to be fixed according to the maximum length of the assumed echoes.

The invention has been defined essentially with reference to linear digital modulation. It can be extended to other types of modulation for which the

10 processing steps proposed would be appropriate, at least in part. Such is the case in particular for allied types of modulation, such as continuous-phase digital frequency modulation, among other frequency shift keyings (or FSKs). The person skilled in the art will understand

15 that, in this case, the modelling of the instantaneous frequency conforms to an expression much like formula E10 on replacing $\Delta f0$ by zero. All the parameters in E10 are in this case real numbers.

$$r(t) = \text{Re}\left[\sum_k a(k) g(t - kT) e^{j2\pi f_0 t}\right] = \text{Re}[z(t)] \qquad \text{(E1)}$$

$$p(t) \rightarrow p(t) e^{j2\pi f_i t} \qquad \text{(E2)}$$

$$e^{j2\pi f_i t} \qquad \text{(E3)}$$

$$x(t) = z(t) z^*(t - \tau) \quad (\tau \text{ small}) \qquad \text{(E4)}$$

$$X(f) \quad \rightarrow \quad X_1(f) = \frac{X(f)}{\text{mean }(|X(f_i)|)} \qquad \text{(E5)}$$

$$f_i \in [f - \Delta f_i, f + \Delta f_i]$$

$$z(t) = \sum_k a(k) g(t - kT) e^{j2\pi \Delta f_0 t} \qquad \text{(E10)}$$

$$z(t) = \sum_k \left(a(k) e^{j2\pi \Delta f_0 kT}\right) \left(g(t - kT) e^{j2\pi \Delta f_0 (t - kT)}\right) \qquad \text{(E11)}$$

$$g_m(t) = g(t) e^{j2\pi \Delta f_0 t} \qquad \text{(E12)}$$

$$a_m(k) = a(k) e^{j2\pi \Delta f_0 kT} \qquad \text{(E13)}$$

$$z(t) = \sum_k a_m(k) g_m(t - kT) \qquad \text{(E14)}$$

$$(a_m(k), g_m(t)) \qquad \text{(E15)}$$

$$\sum_k c_m(k) h_m(t - kT) = \sum_k a_m(k) g_m(t - kT) \qquad \text{(E18)}$$

$$z(t) = \sum_k c_m(k) h_m(t - kT) \qquad \text{(E20)}$$

$$g_m(t) = \sum_{i=0}^{n} \alpha_i h_m(t - iT) \qquad \text{(E21)}$$

$$\begin{cases} z(t) = \sum_k c_m(k) h_m(t - kT) \\ c_m(k) = \sum_{i=1}^{n} \alpha_i a_m(k - i) \end{cases} \qquad \text{(E22) (E23)}$$

$$\begin{cases} \varphi_1(t) = \underset{\left[0,\frac{T}{2}\right[}{\text{rect}}(t) \\ \forall i \in \{1,..b\} \quad \varphi_i(t) = \underset{\left[(i-1)\frac{T}{2},i\frac{T}{2}\right[}{\text{rect}}(t) = \varphi_1\left(t-(i-1)\frac{T}{2}\right) \end{cases}$$

(E25) (E26)

$$\begin{cases} z_p(t) = \sum_k c_m(k)h_{pm}(t-kT) \\ c_m(k) = \sum_{i=1}^{n} \alpha_i\, a_m(k-i) \end{cases}$$

(E30)

$$\begin{cases} \varphi_1(t) = \underset{\left[0,\frac{T}{q}\right[}{\text{rect}}(t) \quad q \in IN \setminus \{0,1\} \\ \forall_i \in \{1,..b\} \quad \varphi_i(t) = \underset{\left[(i-1)\frac{T}{q},i\frac{T}{q}\right[}{\text{rect}}(t) \end{cases}$$

(E34) (E35)

$$z_p(t) = \sum_k z(k).\varphi_1\left(t-(k-1)\frac{T}{2}\right)$$

(E36)

$$h_{pm}(t) = h_1.\varphi_1(t) + h_2.\varphi_1\left(t-\frac{T}{2}\right) + ...h_{2L}.\varphi_1\left(t-(2L-1)\frac{T}{2}\right)$$

(E37)

$$\forall k \begin{cases} z(2k+1) = c_m(k)h_1 + c_m(k-1)h_3 + ... + c_m(k-L+1)h_{2L-1} \\ z(2k+2) = c_m(k)h_2 + c_m(k-1)h_4 + ... + c_m(k-L+1)h_{2L} \end{cases}$$

(E38)

$$Z_{L1}(k) = \begin{bmatrix} z(2k+1) \\ z(2k+2) \\ . \\ . \\ . \\ . \\ z(2k+2L_1-1) \\ z(2k+2L_1) \end{bmatrix}$$

(E41)

$$\underset{(2L_1 \times 2L_1)}{M_{L1}} = \sum_k Z_{L_1}(k).Z^*_{L_1}(k)$$

(E42)

$$\underset{(2L \times 2L)}{M_L} = \sum_k Z_L(k) . Z_L^*(k) \qquad \text{(E43)}$$

$$Z_L(k) = \begin{bmatrix} z(2k+1) \\ z(2k+2) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ z(2k+2L-1) \\ z(2k+2L) \end{bmatrix} \qquad \text{(E44)}$$

$$V = \begin{bmatrix} +h_{2L}^* \\ -h_{2L-1}^* \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ +h_2^* \\ -h_1^* \end{bmatrix} \qquad \text{(E45)}$$

$$
\begin{aligned}
\forall k \, z_{L,k}(t) = \quad & c_m(k-L+1) . Z_1(t) \\
+ \quad & c_m(k-L+2) . Z_2(t) \\
+ \quad & \ldots \ldots \\
+ \quad & c_m(k) \quad . \quad Z_L(t) \qquad \text{(E47)} \\
+ \quad & \ldots \ldots \\
+ \quad & c_m\left(k+L_1\right) . \ Z_{L+L_1-1}(t)
\end{aligned}
$$

$$Z_1(t) = h_{pm}\left(t-(-L+1)T\right) \cdot \underset{[0,L_1 T[}{\text{rect}(t)}$$

$$Z_2(t) = h_{pm}\left(t-(-L+2)T\right) \cdot \underset{[0,L_1 T[}{\text{rect}(t)}$$

......

$$Z_L(t) = h_{pm}(t) \cdot \underset{[0,L_1 T[}{\text{rect}(t)} \tag{E48}$$

......

$$Z_{L+L_1-1}(t) = h_{pm}\left(t-L_1 T\right) \cdot \underset{[0,L_1 T[}{\text{rect}(t)}$$

$$\forall k \begin{cases} z(qk+1) = c_m(k)h_1 + c_m(k-1)h_{q+1} + ...c_m(k-L+1)h_{(q-1)L+1} \\ z(qk+2) = c_m(k)h_2 + c_m(k-1)h_{q+2} + ...c_m(k-L+1)h_{(q-1)L+2} \\ .... \\ z(qk+q) = c_m(k)h_q + c_m(k-1)h_{q+q} + ...c_m(k-L+1)h_{qL} \end{cases} \tag{E50}$$

$$Z_{L_1}(k) = \begin{bmatrix} z(qk+1) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ z(qk+qL_1) \end{bmatrix} \tag{E51}$$

$$\underset{(q.L_1 \times q.L_1)}{M_{L_1}} = \sum_k Z_{L_1}(k).Z_{L_1}^*(k) \tag{E52}$$

$$\underset{(q.L_1 \times 1)}{V_i} = \begin{bmatrix} v_i(1) \\ \cdot \\ \cdot \\ v_i(q.L_1) \end{bmatrix} \quad i \in \{L_1+L, ..., qL_1\} \tag{E55}$$

$$P_i \atop (q.L \times q.(L_1 + L) - 1)} =$$

$$
\begin{bmatrix}
\begin{bmatrix} v_i(qL_1 - q + 1) \\ \cdot \\ \cdot \\ v_i(qL_1) \\ 0 \\ \\ 0 \end{bmatrix}
&
\begin{bmatrix} v_i(qL_1 - 2q + 1) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ v_i(qL_1) \\ 0 \end{bmatrix}
& \begin{matrix} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{matrix}
& \begin{matrix} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{matrix}
& \begin{matrix} 0 \\ \\ \begin{bmatrix} v_i(1) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ v_i(qL_1 - q) \end{bmatrix} \end{matrix}
& \begin{matrix} 0 \\ \\ 0 \\ \\ \begin{bmatrix} v_i(1) \\ \cdot \\ \cdot \\ v_i(q) \end{bmatrix} \end{matrix}
\end{bmatrix}
\qquad \text{(E56)}
$$

$$G = \sum_i P_i . P_i^* \qquad \text{(E57)}$$

$$
h_{pm} \atop (q.L \times 1)} =
\begin{bmatrix} h_1 \\ h_2 \\ \cdot \\ \cdot \\ \cdot \\ h_{qL} \end{bmatrix}
\qquad \text{(E58)}
$$

$$H = \begin{bmatrix} h\big((L-1).q+1\big) & h\big((L-2).q+1\big) & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ h(L.q) & h\big((L-1).q\big) & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & h(L.q) & \cdot & \cdot \\ \cdot & \cdot & \cdot & h(1) \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & h(q) \end{bmatrix}$$

$$\underset{\big(q.Lx(2L-1)\big)}{}$$

(E63)

$$\big(H^{*}.H\big)^{-1} H^{*}$$
$$\big[(2L-1)_{\times qL}\big]$$

(E64)

$$E\big(a(k)\,a(k+p)^{*}\big) \approx \sum_{k} a(k)\,a(k+p)^{*} = \begin{vmatrix} 1 & \text{for } p=0 \\ 0 & \text{otherwise} \end{vmatrix}$$

(E70)

$$P(Z) = \sum_{i=0}^{N} \alpha_{i} \, Z^{-i}$$

(E71)

$$P_{2}(Z) = P(Z).P^{*}(Z) = \left(\sum_{i=0}^{N} \alpha_{i} \, Z^{-i}\right)\left(\sum_{j=0}^{N} \alpha_{j}^{*} \, Z^{+j}\right)$$

(E72)

$$Q(Z) = \sum_{r=-N}^{r=+N} \left(\sum_{K}\big[c_{m}(k)\,c_{m}^{*}(k+r)\big]\right) Z^{-r}$$

(E73)

$$P(Z) = K_{1}.\prod_{i=1}^{N}\big(1-\beta_{i}\,Z^{-i}\big)$$

(E75)

$$P_{2}(Z) = K_{1}.K_{1}^{*}\prod_{i=1}^{N}\big(1-\beta_{i}\,Z^{-i}\big)\big(1-\beta_{i}^{*}\,Z^{-i}\big)$$

(E76)

$$V = \frac{1}{nb}\sum_{k}\left[\big|a_{m}(k)\big| - \frac{1}{nb}\sum_{k}\big|a_{m}(k)\big|\right]^{2}$$

(E80)

$$E = \left(a(k)a(k+p)^{\bullet}\right) = \begin{vmatrix} \lambda_p & \text{arbitrary for} & -e \le p \le e & \left(e \in IN \text{ and small}\right) \\ 0 & \text{otherwise} \end{vmatrix} \tag{E81}$$

with e a not overly large integer

$$d(x,y) = \frac{1}{kx}\sum_{k=1}^{kx}\frac{1}{\sigma}\frac{1}{\sqrt{2\Pi}}e^{-\frac{|x+jy-a(k)|^2}{2\sigma^2}} \tag{E82}$$

σ is a constant whose value has been fixed by simulation

$$d(x,y) = \frac{1}{nb}\sum_{k=1}^{nb}\frac{1}{\sigma}\frac{1}{\sqrt{2\Pi}}e^{-\frac{|x+jy-a(k)|^2}{2\sigma^2}} \tag{E82}$$

$$\int_{x,y} d^2(x,y)dx\,dy \tag{E85}$$

$$\int d_a(x,y)dxdy < \frac{1}{n-1}\left(\int d(x,y)dxdy - \int d_a(x,y)dxdy\right).\lambda \tag{E86}$$

$$e(k) = \overset{Min}{e_i}\left(\left\|a_{(k)} - e_i\right\|^2\right) \quad . \tag{E87}$$

$e_i \in \{e_1, e_2, ...\}$ set estimated at 900

$$z(t) = \sum_k e_k \, g(t - kT) e^{j2\pi f_o t}$$

(e_k) is the estimated symbol train

$\dfrac{1}{T}$ is the estimated tempo

$$g(t) \equiv \left(\sum_i \alpha_i \, h_{pm}(t - iT) \right) e^{j2\pi f_o t}$$

h_{pm} is the estimated function of smallest support

(α_i) is the estimated filtering residual

$f_0 = f_i + \Delta f_0$ is the estimated carrier (E91)

$$\frac{1}{b/2} \int_0^{b/2 T} \| r(t) - z(t) \|^2 \, dt \qquad (E92)$$